EXERCISES.

19

SHOW geometrically (without using the calculus) that the asymptote of the hyperbolic spiral, $r = a : \theta$, is parallel to the initial line and distant a above it.

[*L. G. Barbour.*]

20

A TUNNEL section, consisting of a rectangle surmounted by a semicircle, is required to accommodate a rectangle of breadth B and height H. Show how to determine its proportions so that its (1) area, (2) perimeter shall be least.

[*William M. Thornton.*]

21

IF THE central force on a body moving in a parabola, latus rectum $4m$, were to cease acting at the vertex and continue interrupted till the body had described an angle of 60° about the focus; determine the orbit it would afterwards describe.

[*William Hoover.*]

22

THE dome of the rotunda of the University of Virginia is spherical. The length of its meridian section is $85^f.2$ and the girth of its base is 214^f. From these data it is required to compute the radius and the surface of the dome.

[*William M. Thornton.*]

23

IF TWO parabolas P and P' cut each other at right angles at a point A on a third parabola P'', all three parabolas having a common focus F, and if the tangent line drawn at A to P'' cut P and P' in B and C, then will one parabola that passes through B and C and has F for its focus cut P'' at right angles.

[*H. A. Newton.*]

24

The curves of the family

$$\left(\frac{C}{r}\right) = \cos(p + n\theta),$$

where n is a parameter, all pass through a fixed point and cut orthogonally the fixed curve

$$\left(\frac{r}{c}\right)^n = \cos n\theta,$$

provided $C^n = \frac{1}{2}c^n \cos p$.

[Generalization of 14.] [*Alfred C. Lane.*]

25

AN elastic ring of radius a is placed gently on a smooth paraboloid of revolution, whose axis is vertical; find, by use of the principle of energy, the lowest position to which the ring will descend, and its position of static equilibrium. [*R. D. Bohannon.*]

NOTES.

3

MR. JOSEPH B. MOTT, of Worthington, Minn., sends, in connexion with an invalid deduction of the logarithmic series, several ingenious combinations for the computation of logarithms of primes. We note the following: —

$$\log 11 = 1 + \frac{1}{2}\log 2 - 2\log 3 + \log 7$$
$$+ M\left\{\frac{1}{19601} + \frac{1}{3}\cdot\frac{1}{19601^3} + \frac{1}{5}\cdot\frac{1}{19601^5} + \ldots\right\}.$$
$$\log 13 = \frac{3}{2}\log 2 + \frac{3}{2}\log 11 - \log 3 - \frac{1}{2}\log 7$$
$$- M\left\{\frac{1}{21295} + \frac{1}{3}\cdot\frac{1}{21295^3} + \frac{1}{5}\cdot\frac{1}{21295^5} + \ldots\right\}.$$

Like results are given for log 17 and log 19; and the logarithms of these and other primes are computed with considerable facility to thirty-two places.

4

PERHAPS no modern geometer has fallen upon an easier and more rapid process for such computations than that indicated by Newton (*Epistola posterior ad Oldenburgium*, Oct. 24, 1676; Opuscula 1, 328). Newton computes for $x = 0{,}1$; $0{,}2$; $0{,}01$; $0{,}02$; $0{,}001$; $0{,}002$ the values of

$$\log\sqrt{\frac{1+x}{1-x}} = x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \ldots,$$
$$\log\sqrt{\frac{1}{1-x^2}} = x^2 + \frac{1}{4}x^4 + \frac{1}{6}x^6 + \ldots;$$

whence by the aid of the simple interpolation formula

$$\log n = \log(n-x) + d\left(1 + \frac{x}{2n} + \frac{x^3}{12n^3} + \ldots\right),$$